Non-Euclidean Electromagnetic Kerr Model for Hydrogen

Abstract

A Balmer series of observed hydrogen data was compared to two geometric levels of atomic theory and modeling. The first theory compared was the Euclidean-based (Minkowski or Lorentz metric) special relativistic Dirac theory, with QED corrections added. The second theory applied was a non-Euclidean electromagnetic (EM) Kerr field theory with Euclidean QED corrections added. Each model was used to predict the Balmer series transition wavelengths, and then compared to the observed wavelength data. The statistics (sample averages, standard deviations) for the model performances were computed, and show a noticeable increase in accuracy and precision of the model predictions using the non-Euclidean EM Kerr field theory with QED, compared to Euclidean Dirac theory with QED. These results suggest Euclidean Dirac theory is too restrictive because of its special geometric nature, and does not incorporate an important "beyond special/non-Euclidean" relativistic contributor. According to non-Euclidean field theory, the time dilation the electron experiences is a function of not only its velocity (as in Dirac theory), but the electron's time dilation is also a function of its position in the "generalized" electromagnetic Kerr field of the hydrogen atom. The time dilation the electron is subjected to is then stronger than in Dirac special relativity, and increases subshell energies. This causes a "compression" of the probabilistic hydrogen subshells towards the proton (on average), even more so than the introduction of special relativity. The modeling of these added non-Euclidean relativistic effects produces predictions in better agreement with the observed hydrogen Balmer data.

The Hydrogen Balmer Data

A set of observed Balmer series hydrogen data was obtained from the text book: *The Physics of Atom and Quanta,* H. Haken, H. Wolf and W. Brewer, 6th Edition, 2004, Springer-Verlag. The data set from the text is:

Table 8.2. The first 20 lines of the Balmer series of hydrogen. The numbers quoted are wavelengths in air, the wavenumbers in vacuum, and the values calculated from the Balmer formula

n	λ_{air} [Å]	$\tilde{\nu}_{vac}$ [cm^{-1}]	$R_H\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$
H_α 3	6562.79	15233.21	15233.00
H_β 4	4861.33	20564.77	20564.55
H_γ 5	4340.46	23032.54	23032.29
H_δ 6	4101.73	24373.07	24372.80
H_ε 7	3970.07	25181.33	25181.08
H_ζ 8	3889.06	25705.84	25705.68
H_η 9	3835.40	26065.53	26065.35
H_ϑ 10	3797.91	26322.80	26322.62
H_ι 11	3770.63	26513.21	26512.97
H_κ 12	3750.15	26658.01	26657.75
H_λ 13	3734.37	26770.65	26770.42
H_μ 14	3721.95	26860.01	26859.82
H_ν 15	3711.98	26932.14	26931.94
H_ξ 16	3703.86	26991.18	26990.97
H_o 17	3697.15	27040.17	27039.89
H_π 18	3691.55	27081.18	27080.88
H_ϱ 19	3686.83	27115.85	27115.58
H_σ 20	3682.82	27145.37	27145.20

The (in vacuo) Balmer series data in this text are in wavenumbers (in cm^{-1}), which were converted to wavelengths in meters, for transitions from main shells $n = 3$ to $n = 2$, then $n = 4$ to $n = 2$, etc., up to $n = 20$ to $n = 2$. (18 data points.) The observed Balmer data are astrophysical, taken using astronomical spectroscopy. Assuming theory is correct, there must have been numerous (n) subshell -to- ($n = 2$) subshell transitions involved in these data. But only a single transition data point is listed for each Balmer transition. A single transition data value was assumed to be obtained from a weighted average, with weights based on the observed subshell-to-subshell transition intensities. Past historical hydrogen lab spectroscopy shows the "humps" seen around a single transition were averaged with intensity weights to provide a single transition value. Or if the spectroscopic machine had relatively poor resolution (but apparently it was pretty good, as will be shown), nature itself weighted the single observed line.

Euclidean Dirac Theory Plus QED Results

The Balmer series is produced by transitions to $n = 2$. Ignoring hyperfine splitting, there are 3 sublevels for $n = 2$, the outer $2P_{3/2}$ ($l = 1$, spin-orbit magnetic) sublevel with QED, then the mid spherical $2S_{1/2}$ ($l = 0$, nonmagnetic, no spin-orbit) sublevel with QED, then the lowest $2P_{1/2}$ ($l = 1$, spin-orbit magnetic) sublevel with QED. In basic Dirac theory, the last two (without QED) are

degenerate. Introducing QED effects/corrections "breaks the degeneracy." The paper at the following link provides the needed theoretical hydrogen subshell energies to use for a Balmer series prediction, with QED corrections:

http://www.nist.gov/data/PDFfiles/jpcrd100.pdf

The above NIST paper lists (p. 853) all of the predicted hydrogen subshell energies (in cm^{-1}) with QED corrections for n = 1, 2, …, 8 and further. It is an older paper (1977), but the paper's predicted values agree well with the current values listed at NIST. Most of the Dirac equations are functions of the fine structure constant α, for which NIST has provided accurate values for many decades. Today's α doesn't differ much from that of 1977. (The prediction equations in this paper presented later used a modern (2013) value of α.)

For the Dirac + QED predictions for the $2P_{3/2}$, $2S_{1/2}$ and $2P_{1/2}$ subshells, the values listed in the paper were used. For the analysis here, the "plus QED" subshell energies also listed in the paper for the Balmer series predictions for n = 3, …, 8 were used. The basic Dirac equation (plus the main QED corrections) was then used to compute predictions for n = 9, 10, … , 20. The basic Dirac equation (without QED) used was eq. (2.4) of the above paper, listed here:

$$E_{n,j} = \mu c^2 \left[1 + \left(\frac{\alpha}{n - \varepsilon} \right)^2 \right]^{-1/2} - \mu c^2 \qquad (0)$$

where $\varepsilon = j + 1/2 - [(j+1/2)^2 - \alpha^2]^{1/2}$ and μ is the electron's reduced rest mass in hydrogen. The main theoretical QED corrections added to (0) for n = 9, …, 20 are given by eq. (13) of this paper.

Given the transition rules $\Delta j = 0, \pm 1, \Delta l = \pm 1$, the allowed transitions to the three n = 2 sublevels are

$nP_{1/2}$ <-> $2S_{1/2}$
$nP_{3/2}$ <-> $2S_{1/2}$
$nS_{1/2}$ <-> $2P_{1/2}$
$nD_{3/2}$ <-> $2P_{1/2}$
$nD_{5/2}$ <-> $2P_{1/2}$
$nS_{1/2}$ <-> $2P_{3/2}$
$nD_{3/2}$ <-> $2P_{3/2}$
$nD_{5/2}$ <-> $2P_{3/2}$

The procedure just described was used to predict the (18) transition wavelengths (using subshell energy differences) for each of the above transitions, using the NIST paper's subshell QED-corrected energy values for n = 2, …, 8, and basic Dirac predictions (plus main QED corrections) for n = 9, …, 20. The predictions were then compared to the observed data. For the $n\mathrm{P}_{1/2}$ <-> $2\mathrm{S}_{1/2}$ transition, the results were:

$n\underline{\mathrm{P}_{1/2} <-> 2\mathrm{S}_{1/2}}$
Sample average prediction error = $+2.67999673 \times 10^{-12}$ m
Sample standard deviation = $6.24440238 \times 10^{-13}$ m
Standard error (the above divided by $\sqrt{18}$) = $1.47181976 \times 10^{-13}$ m
5-sigma confidence interval = ($+1.94408685 \times 10^{-12}$, $+3.41590661 \times 10^{-12}$) m

For this analysis, the individual observed, predicted, and prediction errors (wavelengths in m) were :

n = 3	6.56460457E-007	6.56458428E-007	+2.02947188E-012
n = 4	4.86268507E-007	4.86265562E-007	+2.94506857E-012
n = 5	4.34168355E-007	4.34165816E-007	+2.53871789E-012
n = 6	4.10288897E-007	4.102868E-007	+2.09692431E-012
n = 7	3.97119612E-007	3.97117175E-007	+2.43736638E-012
n = 8	3.89016659E-007	3.89012797E-007	+3.86230807E-012
n = 9	3.83648443E-007	3.83644966E-007	+3.47726963E-012
n = 10	3.79898795E-007	3.7989538E-007	+3.41463844E-012
n = 11	3.77170475E-007	3.7716795E-007	+2.52416634E-012
n = 12	3.75121774E-007	3.75119596E-007	+2.17801972E-012
n = 13	3.73543414E-007	3.73540825E-007	+2.58900413E-012
n = 14	3.7230068E-007	3.72297544E-007	+3.13686476E-012
n = 15	3.7130358E-007	3.71300543E-007	+3.03669394E-012
n = 16	3.70491398E-007	3.70488537E-007	+2.86076901E-012
n = 17	3.6982016E-007	3.69818252E-007	+1.9077457E-012
n = 18	3.69260128E-007	3.69258413E-007	+1.71552799E-012
n = 19	3.68787997E-007	3.68785943E-007	+2.05362119E-012
n = 20	3.68386948E-007	3.68383512E-007	+3.43576319E-012

The observed Balmer data (converted from the wavenumbers in the referenced text book) are in the first data column, the predictions are in the second column, and the differences (errors) are in the third (last) column. The magnitudes of the prediction errors (observed wavelength minus predicted wavelength) are ~10^{-12} m. The transition wavelengths themselves are ~10^{-7} m for the Balmer series, so the prediction errors are about 5 orders of magnitude smaller than the observed variable magnitude. This indicates both the model and the data are reasonably accurate and precise. The precision in the fit even allows for an evaluation of a/this model's bias (inaccuracy).

For the above, all 18 prediction errors were positive, indicating a model bias. The positive prediction errors say this analysis produced predicted transition wavelengths that are too short as

compared to "the truth" given by the data. Some type of bias is to be expected, since the observed data involved more transitions than just this one. Using only this single transition, this "not complete" model represents a "too inflated" hydrogen atom relative to the "correct compression" manifest in the observed data. The predicted wavelengths were too small (observed > predicted, positive error) and too energetic, saying the (*n*)subshell-to-(2)subshell transition delta-energies were too large and "inflated" compared to the data. The 5-sigma confidence interval for the true mean prediction error also does not contain zero, indicating a model bias. As mentioned, this is to be expected, since the observed data undoubtedly contain other transitions of greater intensity than does this transition.

The analysis was repeated for the other allowed Balmer transitions, producing the following results (values in m):

Transition	Sample Average Prediction Error	Sample Standard Deviation
$nP_{1/2}$ <-> $2S_{1/2}$	$+2.67999673 \times 10^{-12}$	$6.24440238 \times 10^{-13}$
$nP_{3/2}$ <-> $2S_{1/2}$	$+3.06225625 \times 10^{-12}$	$1.12816117 \times 10^{-12}$
$nS_{1/2}$ <-> $2P_{1/2}$	$+3.3067649 \times 10^{-12}$	$6.44122003 \times 10^{-13}$
$nD_{3/2}$ <-> $2P_{1/2}$	$+3.65092165 \times 10^{-12}$	$1.34348622 \times 10^{-12}$
$nD_{5/2}$ <-> $2P_{1/2}$	$+3.77833809 \times 10^{-12}$	$1.67893344 \times 10^{-12}$
$nS_{1/2}$ <-> $2P_{3/2}$	$-2.80327764 \times 10^{-12}$	$2.44166176 \times 10^{-12}$
$nD_{3/2}$ <-> $2P_{3/2}$	$-2.45910605 \times 10^{-12}$	$1.50638804 \times 10^{-12}$
$nD_{5/2}$ <-> $2P_{3/2}$	$-2.33168411 \times 10^{-12}$	$1.18493865 \times 10^{-12}$

For each case with a positive average, all prediction errors were also positive. For those with a negative average, all prediction errors were negative. A negative result describes a model that is "too compressed" relative to the data. The predicted wavelengths are too large (observed < predicted, negative errors), relating back to a too small subshell-to-subshell delta-energy difference. This "too compressed" bias as compared to the data is also to be expected for these particular single transition models.

An observed data value comes from a blend of all of these transitions, with the value of the single shell-to-shell value equal to a transition-intensity weighted average. The predicted weights could be computed to produce a single shell predicted value, and a better single shell prediction obtained to compare to the data. But, for comparison to the next atomic model, seeing the Euclidean Dirac + QED results for these separated-out transitions will be informative.

Non-Euclidean Eelectromagnetic Kerr Theory Results

The next atomic model used to compare to the hydrogen data is based on non-Euclidean spacetime differential geometries. The idea is to parameterize an atomic sized non-Euclidean field to represent the electrostatic field generated by (and surrounding) the proton. The first non-Euclidean atomic-sized model is spherical, with a Schwarzschild metric as the special case.

For spherical subshells (n, $l = 0$, $j = ½$), the time dilation the electron experiences in this (special case Schwarzschild) spherical non-Euclidean field theory, along an equatorial circular orbit, is given as

$$\frac{dt}{d\tau} = \left(1 - \left(\frac{v_n}{c}\right)^2 - \frac{r_S}{r_n}\right)^{-1/2} \tag{1}$$

where the electronic Schwarschild radius (defined by the extended gravitoelectromagnetic Equivalence Principle) is $r_S = \left|e_e e_p\right| / 2\pi\varepsilon_0 m_e c^2 = 2\alpha\hbar / m_e c$ where m_e is the electron's rest mass, not its reduced rest mass. But as in Euclidean Dirac theory, the reduced rest mass μ of the electron must be used in the energy equations. (Please see the Appendix for a derivation of the electronic Schwarzschild radius r_S.) Equation (1) is a special case of (5), with a Schwarzschild form of (4).) The time dilation given by (1) is only electronic in form, per the definition of r_S, which also shows it is only defined for a system of two charges (and conforms to muonic hydrogen, e.g.). Note the mass and charge of the electron themselves (and the charge of the proton) set the basic non-Euclidean Schwarzschild structure of the field within which the electron orbits. Hence, if the mass of the orbiting body changes, for example, like in muonic hydrogen, the entire metric structure shifts to accommodate the new two-body system. A main effect of using a non-Euclidean field theory is the introduction of the position-dependent field contribution to the total time dilation, with the necessary inclusion of the r_S / r_n term. The electron's time dilation is hence greater than in special relativity.

The Schwarzschild electron orbital radius and velocity (r_n and v_n) in the n^{th} spherical subshell (needed in the time dilation (1)) are initially given by

$$\begin{aligned} r_n &= r_B \frac{d\tau}{dt} \\ v_n &= v_B \left(\frac{dt}{d\tau}\right)^{1/2} \end{aligned} \tag{2}$$

where r_B and v_B are the nonrelativistic Bohr radius and velocity for the respective n^{th} spherical main shell. Using Bohr values provides only initial approximate values for the needed

Schwarzschild r_n and v_n and the non-Euclidean time dilation. Due to the nonlinear nature of non-Euclidean field theory, numerical iterations are useful, and sometimes required. Double precision numerical iteration to convergence provides essentially theoretically exact results. To compute a correct value of the Schwarzschild time dilation for a given n, the values of r_n and v_n are initialized at their Bohr values in eq. (2) (initialize the time dilation to 1), and then the time dilation (eq. (1)) is updated. Next, the updated time dilation is used to compute new Schwarzschild relativistic values of r_n and v_n, per (2). These updated radius and velocity values are then reinserted into the time dilation eq. (1), and the iteration continued until convergence. Note therefore, time dilation is quantized, as it is in Euclidean Dirac theory.

Given the converged values of orbital radius, velocity and time dilation, the non-Euclidean electronic spherical (Schwarzschild) total orbital energy the electron experiences is given as

$$E_n = \frac{\mu c^2}{2}\left(\left[\left(1-\frac{r_S}{r_n}\right)\frac{dt}{d\tau}\right]^2 - 1\right) \tag{3}$$

where now the electron's reduced rest mass μ resides in this equation. While equation (3) looks completely gravitational in identity (allowed by the extended Gravitoelectromagnetic Equivalence Principle) it is in fact completely electronic in form, and specific to a system of bound charges (here, hydrogen). Nowhere in this equation does Newton's gravitational constant G appear.

These equations are specific for the exact spherical symmetry of a Schwarzschild geometry, so while not explicitly in the equations, the Dirac quantum numbers are j = ½ and l = 0 (true S subshell). The Dirac quantum number s is also unimportant here. No electron-spin-magnetic effects are present due to the complete spherical symmetry of a Schwarzschild field.

To incorporate non-Euclidean spin-orbit magnetic effects, the differential geometry can be generalized to that of Kerr. (In the gravitational world, this predicts gravitomagnetism, as was successfully observed by Gravity Probe B in orbit about the spinning mass of the Earth.) The extended Gravitoelectromagnetic (GEM) Equivalence Principle allows electromagnetism to be represented as a type of non-Euclidean electromagnetic Kerr "frame dragging," which suggests a "unification" of "electro-" and "gravito-" magnetisms.

The electromagnetic Kerr 4 x 4 timelike metric tensor, in spherical polar spacetime coordinates $\mathbf{x} = (r,\theta,\phi,t)^T$, is (dropping orbital indexing for now):

$$\mathbf{G} = \begin{bmatrix} g_{rr} & 0 & 0 & 0 \\ 0 & g_{\theta\theta} & 0 & 0 \\ 0 & 0 & g_{\phi\phi} & g_{\phi t} \\ 0 & 0 & g_{t\phi} & g_{tt} \end{bmatrix} \tag{4}$$

where

$$g_{rr} = -\frac{\Sigma}{c^2 \Delta}, \quad g_{\theta\theta} = -\frac{\Sigma}{c^2}, \quad g_{\phi\phi} = -\frac{1}{c^2}\left[\frac{r^2 + a^{2\ 2} - \Delta a^2 \sin^2\theta}{\Sigma}\right]\sin^2\theta,$$

$$g_{tt} = \frac{\Delta - a^2 \sin^2\theta}{\Sigma}, \quad \text{and} \quad g_{t\phi} = g_{\phi t} = \frac{1}{c^2}\left[\frac{a \sin^2\theta \quad r^2 + a^2 - \Delta}{\Sigma}\right]$$

with

$$\Sigma = r^2 + a^2 \cos^2\theta, \quad \text{and} \quad \Delta = r^2 + a^2 - r_S r$$

Along with the electronic Schwarzschild r_S, the electromagnetic Kerr "frame dragging parameter *a*" enters the equations The final forms of these equations are completely electromagnetic. Newton's *G* does not appear anywhere. It should also be stressed, especially so here, the "magnetism" modeled here is "central" in that it is accorded to the "spin of a central body," as easily modeled with non-Euclidean field theory. This is the analog in Euclidean theory, where the electron's current-loop orbital motion sets up an "effective centrally located" magnetic N-S dipole, creating the "central" magnetic field within which the electron orbits. This is also the basis for the spin-orbit interaction. Here, in the non-Euclidean field theory, the centrality of the dominant magnetism is also maintained, with the frame dragging parameter *a* defined as proportional to the "spin" of a "central body."

For any generalized 4-D spacetime geometry, the generalized time dilation equals

$$\frac{dt}{d\tau} = \left(\frac{d\mathbf{x}^T}{dt}\mathbf{G}\frac{d\mathbf{x}}{dt}\right)^{-1/2} \tag{5}$$

(Equation (5) is obtained by rewriting the usual form of the metric with only $d\tau^2$ on the left.) Expansion of the quadratic form for a Kerr geometry shows, for equatorial circular obits, the Kerr electromagnetic time dilation simplifies to

$$\frac{dt}{d\tau} = \left(1 - \left(\frac{v_n}{c}\right)^2 \left(1 + \frac{a_n^2}{r_n^2} + \frac{r_S a_n^2}{r_n^3}\right) - \frac{r_s}{r_n} + 2\frac{v_n}{c^2}\frac{a_n r_S}{r_n^2}\right)^{-1/2} \tag{6}$$

where subindexing by the main shell number *n* has been reintroduced. The correct value of a_n is to be determined shortly.

At this point, the rest of the full set of Dirac quantum numbers *n*, *j*, and *l* are inserted into in the equations as follows. The principle magnetic interaction in hydrogen is the spin-orbit magnetic interaction, the value of which is "directed" by the set of Dirac quantum numbers. The complete Euclidean Dirac theory which incorporates Born's probability, yields the degenerate Dirac eq. (2.4) of the NIST paper (eq. (0) of this paper). For the "interior" subshell-to-subshell energy differences within a main shell (for example, for the $2P_{3/2}$ <-> $2S_{1/2}$ transition), this equation can provide/predict, the Euclidean Dirac spin-orbit electron-magnetic-moment-orientation delta-energy values existing between these "interior" subshells. These predicted spin-orbit delta energies then have all of the Born spin-orbit-increasing probabilistic effects (averaging) included. Hence, for predictions, set

$$\Delta E_{so/n,j} = E_{D_{n,j}} - E_{D_{n,1/2}} \tag{7}$$

The delta-energy on the left is the correct Dirac spin-orbit value to introduce for a main shell *n* with subshell *j*. The energy $E_{D_{n,j}}$ is the Euclidean Dirac energy eq. (0). When *j* = ½, $\Delta E_{so/n,j} = 0$, and no spin-orbit interaction exists. When *j is* not ½ (e.g., *j* = ⅔), $\Delta E_{so/n,j}$ provides the correct (Dirac/Born probability effected) spin-orbit value as differenced from the spherical *j* = ½ subshell for a main *n*.

Equation (7) can be used to introduce the magnetic quantum number *j* into the non-Euclidean theory. In non-Euclidean field theory, the only way to introduce magnetism is using Kerr frame dragging, and the electromagnetic Kerr total orbital energy shows the route. This energy is generally (for all orbits, not just circular equatorial)

$$E_n = \frac{\mu c^2}{2}\left[\left(\left(1-\frac{r_S r_n}{\Sigma}\right)\frac{dt}{d\tau}+\frac{r_S a_n r_n \sin^2\theta}{\Sigma}\frac{d\phi}{d\tau}\right)^2 -1\right] \tag{8}$$

An important character of the theoretical structure of the Kerr energy equation is its perturbation nature, right in the equation. The aspherical magnetic term involving a is simply added to the spherical Schwarzschild term.

For equatorial circular orbits, the Kerr energy equation simplifies to

$$E_n = \frac{\mu c^2}{2}\left[\left(\left(1-\frac{r_S}{r_n}\right)\frac{dt}{d\tau}+r_S a_n \frac{v_n}{r_n^2}\frac{dt}{d\tau}\right)^2 -1\right] \tag{9}$$

An expansion of (9) separates the energy into individual nonmagnetic and magnetic terms. An isolation of the terms involving the magnetic spin-orbit a_n produces

$$\Delta E_{so} = \mu c^2\left(1-\frac{r_S}{r_n}\right)\left(\frac{dt}{d\tau}\right)^2 \frac{r_S v_n}{r_n^2}a_n + 2\mu c^2\left(\frac{r_S v_n}{2r_n^2}\right)^2\left(\frac{dt}{d\tau}\right)^2 a_n^2 \tag{10}$$

Please note, while not subindexed as so, the quantized time dilation is also a function of n. Also, the r_n and v_n are now even more "general" than in the Schwarzschild theory; they are now the quantized Kerr orbital radii and velocities. Computational results for hydrogen show, though, the converged-upon Kerr radii and velocities are very nearly equal to their Schwarzschild values.

The extended GEM Equivalence Principle allows setting equality between equations (7) and (10). Setting these equations equal results in

$$\begin{aligned} \Delta E_{so/n,j} &= E_{D_{n,j}} - E_{D_{n,1/2}} \\ E_{D_{n,j}} - E_{D_{n,1/2}} &= \mu c^2\left(1-\frac{r_S}{r_n}\right)\left(\frac{dt}{d\tau}\right)^2 \frac{r_S v_n}{r_n^2}a_n + 2\mu c^2\left(\frac{r_S v_n}{2r_n^2}\right)^2\left(\frac{dt}{d\tau}\right)^2 a_n^2 \end{aligned} \tag{11}$$

Equation (11) is a parabola in a_n, and has the Dirac quantum numbers n, j and s (s always ½) correctly incorporated. The solution for a_n is found by root taking. Set

$$\beta_2 = 2\mu c^2 \left(\frac{r_S v_n}{2r_n^2} \right)^2 \left(\frac{dt}{d\tau} \right)^2$$

$$\beta_1 = \mu c^2 \left(1 - \frac{r_S}{r_n} \right) \left(\frac{dt}{d\tau} \right)^2 \frac{r_S v_n}{r_n^2}$$

$$\beta_0 = -(E_{D_{n,j}} - E_{D_{n,1/2}})$$

The parabola is now

$$\beta_2 a_n^2 + \beta_1 a_n + \beta_0 = 0$$

so that

$$a_{n,j,s} = \frac{-\beta_1 + \sqrt{\beta_1^2 - 4\beta_2\beta_0}}{2\beta_2} \tag{12}$$

Numerical computations show the positive root has the physically correct sign. The set of Dirac quantum numbers now reside in the quantized electromagnetic Kerr total orbital energies. The degree of Kerr asphericity needed to model atomic spin-orbit effects is very slight. Hydrogen's r_S equals about 10^{-15} m. A typical $a_{n,j,s}$ value is about +/- 10^{-23} m.

When j = ½, $a_{n,j,s}$ equals zero (magnetic $\beta_0 = 0$). The field theory then drops to the nonmagnetic Schwarzschild (spherical) field theory as the special case. When j is not ½, the two electromagnetic "characteristic lengths" r_S and $a_{n,j,s}$ (devoid of G) enter the Kerr metric, and completely specify the structure of the Kerr electromagnetic spacetime for given values of the Dirac quantum numbers. Note this model is adaptive in the sense each sublevel has its own electromagnetic Kerr metric structure, as $a_{n,j,s}$ (the frame dragging, i.e., the spin-orbit magnetism) discretely changes from subshell to subshell. As in the Schwarzschild case, the added nonlinear field effects change the time dilation. The frame dragging parameter $a_{n,j,s}$ must enter the time dilation iteration, and then both converge on their correct Kerr values, now with all Dirac quantum numbers directing the convergence.

Euclidean QED effects can be inserted into the non-Euclidean theory by adding to β_0, which then introduces explicit functionality on the quantum number l. The dominant electron self

energy and vacuum polarization effects, plus additional spin-orbit effects where appropriate (l not 0, j not ½), were included by computing QED delta-energies using eqs. (2.10) and (2.11) of the NIST paper. These were combined as

$$\Delta E_{QED} = \frac{4\alpha(Z\alpha)^4 m_e c^2}{3\pi n^3}\left(\frac{\mu}{m_e}\right)^3\left[\left(\log\frac{1}{(Z\alpha)^2}+\log\frac{m_e}{\mu}+\frac{11}{24}+\frac{3}{8}-\frac{1}{5}+L_n\right)\delta_{l0}+\left(\frac{3}{8}\frac{C_{lj}}{2l+1}+L_n\right)(1-\delta_{l0})\right] \qquad (13)$$

For hydrogen, $Z = 1$. The Dirac delta function is: $\delta_{l0} = 1$ if $l = 0$ and $\delta_{l0} = 0$ if l not zero. The correct Bethe logarithm value of L_n was obtained from Appendix B of the NIST paper (p. 846). The C_{lj} coefficients were obtained from

$$C_{lj} = \begin{pmatrix} 1/(l+1) & \text{for } j = l+1/2 \\ -1/l & \text{for } j = l-1/2 \end{pmatrix}$$

To incorporate these Euclidean QED effects, eq. (13)'s value was added to the frame dragging β_0 as

$$\beta_0 = -(\Delta E_{so} + \Delta E_{QED})$$

where ΔE_{so} is given by (7). Even when $l = 0$, so there is no spin-orbit interaction, QED effects are still present. These force convergence to an aspherical Kerr structure, even for the "truly spherical" QED-effected $n\mathrm{S}_{1/2}$ subshells in Euclidean Dirac theory. In this non-Euclidean theory, the only way to model aspherical (non-S) subshells is to introduce frame dragging, using the more general magnetic Kerr theory. In this atomic EM Kerr theory, there are no "truly spherical" subshells, where QED effects must necessarily enter as a type of "magnetic frame dragging" in an atomic Kerr field.

The combined Dirac/EM Kerr theory predicts the correct observed subshell-to-subshell transition wavelengths, within a main shell, such as the observed Lamb shift for the $2\mathrm{S}_{1/2}$ <-> $2\mathrm{P}_{1/2}$ transition, a value of about 1057.9 MHz. The theory certainly should predict well, since the known Euclidean Dirac spin-orbit and QED equations (which work) were used to perturb the Kerr theory (through the use of a frame dragging "magnetic Kerr QED perturbation"). The next theoretical route logically is to extend the Euclidean QED effects into the non-Euclideean domain.

The important new result is that this new theory predicts there exists stronger non-Euclidean time dilation effects inside of the whole hydrogen atom, as compared to Euclidean Dirac theory. The stronger time dilation effects "compress" the overall atom more so than in Euclidean special

relativity. The enhanced compression translates into closer, less energetic subshell-to-subshell transitions, with longer transition wavelengths. If these added non-Euclidean time dilation effects exist in nature, the EM Kerr theory predictions should generally agree with the observed data better than Euclidean Dirac + QED.

The EM Kerr field theory, plus Euclidean Dirac spin-orbit and QED effects for interior main shell (subshell) energy shifts, was used to compute subshell energies and energy differences, and non-Euclidean Kerr predicted transition wavelengths. The results for the various non-Euclidean Kerr transitions were:

Transition	Sample Average Prediction Error	Sample Standard Deviation
$n\mathrm{P}_{1/2}$ <-> $2\mathrm{S}_{1/2}$	-1.83537926 x 10^{-12}	1.13719404 x 10^{-12}
$n\mathrm{P}_{3/2}$ <-> $2\mathrm{S}_{1/2}$	-1.4531095 x 10^{-12}	6.34519538 x 10^{-13}
$n\mathrm{S}_{1/2}$ <-> $2\mathrm{P}_{1/2}$	-7.90452013 x 10^{-13}	7.40603382 x 10^{-13}
$n\mathrm{D}_{3/2}$ <-> $2\mathrm{P}_{1/2}$	-4.44336372 x 10^{-13}	8.30573092 x 10^{-13}
$n\mathrm{D}_{5/2}$ <-> $2\mathrm{P}_{1/2}$	-3.16916630 x 10^{-13}	1.11436804 x 10^{-12}
$n\mathrm{S}_{1/2}$ <-> $2\mathrm{P}_{3/2}$	-6.90053527 x 10^{-12}	3.07413769 x 10^{-12}
$n\mathrm{D}_{3/2}$ <-> $2\mathrm{P}_{3/2}$	-6.55440473 x 10^{-12}	2.12330628 x 10^{-12}
$n\mathrm{D}_{5/2}$ <-> $2\mathrm{P}_{3/2}$	-6.42697949 x 10^{-12}	1.78363225 x 10^{-12}

For the first five (dominant) transitions listed, comparison with the Euclidean Dirac + QED results show these non-Euclidean models fair noticeably better. The sample average prediction errors and standard deviations are generally smaller, indicating an increase in accuracy and precision.

The “single transition model” that dominates in the observed Balmer data should have the smallest average prediction error. If the EM Kerr results are sorted according to their average prediction error magnitudes (smallest to largest, average error on the right), the results are

$n\mathrm{D}_{5/2}$ <-> $2\mathrm{P}_{1/2}$	-3.16916630 x 10^{-13}
$n\mathrm{D}_{3/2}$ <-> $2\mathrm{P}_{1/2}$	-4.44336372 x 10^{-13}
$n\mathrm{S}_{1/2}$ <-> $2\mathrm{P}_{1/2}$	-7.90452013 x 10^{-13}
$n\mathrm{P}_{3/2}$ <-> $2\mathrm{S}_{1/2}$	-1.4531095 x 10^{-12}
$n\mathrm{P}_{1/2}$ <-> $2\mathrm{S}_{1/2}$	-1.83537926 x 10^{-12}
$n\mathrm{D}_{5/2}$ <-> $2\mathrm{P}_{3/2}$	-6.42697949 x 10^{-12}
$n\mathrm{D}_{3/2}$ <-> $2\mathrm{P}_{3/2}$	-6.55440473 x 10^{-12}
$n\mathrm{S}_{1/2}$ <-> $2\mathrm{P}_{3/2}$	-6.90053527 x 10^{-12}

In the natural Balmer series, those transitions with higher transition frequencies have higher probabilities of occurring, and theoretically dominated in the data. An ordering of the Balmer transitions by frequency, and hence intensity and probability of occurrence, according to theory (both Euclidean Dirac and non-Euclidean Kerr) is

$n\mathrm{D}_{5/2} - 2\mathrm{P}_{1/2}$
$n\mathrm{D}_{3/2} - 2\mathrm{P}_{1/2}$
$n\mathrm{P}_{3/2} - 2\mathrm{S}_{1/2}$
$n\mathrm{S}_{1/2} - 2\mathrm{P}_{1/2}$
$n\mathrm{P}_{1/2} - 2\mathrm{S}_{1/2}$
$n\mathrm{D}_{5/2} - 2\mathrm{P}_{3/2}$
$n\mathrm{D}_{3/2} - 2\mathrm{P}_{3/2}$
$n\mathrm{S}_{1/2} - 2\mathrm{P}_{3/2}$

For the EM Kerr analysis, the $n\mathrm{P}_{3/2} - 2\mathrm{S}_{1/2}$ and $n\mathrm{S}_{1/2} - 2\mathrm{P}_{1/2}$ transitions are flipped in order, but other than that, the agreement with the EM Kerr ordering is good. The ordering of transitions based on the Euclidean Dirac average prediction error magnitudes (smallest to largest) is

$n\mathrm{D}_{5/2}$ <-> $2\mathrm{P}_{3/2}$	$-2.33168411 \times 10^{-12}$
$n\mathrm{D}_{3/2}$ <-> $2\mathrm{P}_{3/2}$	$-2.45910605 \times 10^{-12}$
$n\mathrm{P}_{1/2}$ <-> $2\mathrm{S}_{1/2}$	$+2.67999673 \times 10^{-12}$
$n\mathrm{S}_{1/2}$ <-> $2\mathrm{P}_{3/2}$	$-2.80327764 \times 10^{-12}$
$n\mathrm{P}_{3/2}$ <-> $2\mathrm{S}_{1/2}$	$+3.06225625 \times 10^{-12}$
$n\mathrm{S}_{1/2}$ <-> $2\mathrm{P}_{1/2}$	$+3.3067649 \times 10^{-12}$
$n\mathrm{D}_{3/2}$ <-> $2\mathrm{P}_{1/2}$	$+3.65092165 \times 10^{-12}$
$n\mathrm{D}_{5/2}$ <-> $2\mathrm{P}_{1/2}$	$+3.77833809 \times 10^{-12}$

The orderings above differ significantly for dominance of transition, even according to Euclidean Dirac theory. The Euclidean Dirac + QED results do not line up well with the frequency/intensity ordering, which is the same for either theory, Kerr or Dirac. The reason for the Dirac theory mismatch is the Euclidean theory does not compress the hydrogen main shells enough. These results show how the overall atomic-sized relativistic structure of the hydrogen atom is in fact not Euclidean. The overall relativistic nature of a hydrogen atom appears to demand a non-Euclidean field modeling.

The listings of the Dirac and Kerr theory prediction errors, shows how almost all removal of model error bias (inaccuracy) is attained by the Kerr model beyond the Dirac model. The dominant transition in the Balmer data should be the $n\mathrm{D}_{5/2} - 2\mathrm{P}_{1/2}$ transition, as predicted by both theories. The Eucldiean Dirac + QED theory produces the following prediction errors:

n = 3 6.56460457E-007 6.56450686E-007 +9.77150608E-012
n = 4 4.86268507E-007 4.86263287E-007 +5.21958415E-012
n = 5 4.34168355E-007 4.34164563E-007 +3.791695E-012
n = 6 4.10288897E-007 4.10285902E-007 +2.99471454E-012
n = 7 3.97119612E-007 3.97116439E-007 +3.17307473E-012
n = 8 3.89016659E-007 3.89012148E-007 +4.51153056E-012
n = 9 3.83648443E-007 3.83644368E-007 +4.07525673E-012
n = 10 3.79898795E-007 3.79894815E-007 +3.98011082E-012
n = 11 3.77170475E-007 3.77167407E-007 +3.06776921E-012
n = 12 3.75121774E-007 3.75119068E-007 +2.70628719E-012
n = 13 3.73543414E-007 3.73540308E-007 +3.10612129E-012
n = 14 3.7230068E-007 3.72297035E-007 +3.64563911E-012
n = 15 3.7130358E-007 3.71300041E-007 +3.53909015E-012
n = 16 3.70491398E-007 3.70488039E-007 +3.35819563E-012
n = 17 3.6982016E-007 3.69817759E-007 +2.40120997E-012
n = 18 3.69260128E-007 3.69257922E-007 +2.20579076E-012
n = 19 3.68787997E-007 3.68785455E-007 +2.54126182E-012
n = 20 3.68386948E-007 3.68383027E-007 +3.92124785E-012

The prediction errors in the right column are all positive, indicating significant model bias (inaccuracy). For the same frequency dominant $n\mathrm{D}_{5/2} - 2\mathrm{P}_{1/2}$ transition, the EM Kerr theory's prediction errors are

n = 3 6.56460457E-007 6.5645706E-007 +3.39726794E-012
n = 4 4.86268507E-007 4.86268287E-007 +2.19322323E-013
n = 5 4.34168355E-007 4.34169072E-007 -7.17467023E-013
n = 6 4.10288897E-007 4.10290159E-007 -1.26206435E-012
n = 7 3.97119612E-007 3.97120544E-007 -9.32065978E-013
n = 8 3.89016659E-007 3.89016153E-007 +5.06071074E-013
n = 9 3.83648443E-007 3.83648304E-007 +1.38985533E-013
n = 10 3.79898795E-007 3.798987E-007 +9.45903717E-014
n = 11 3.77170475E-007 3.77171254E-007 -7.79518956E-013
n = 12 3.75121774E-007 3.75122885E-007 -1.11145104E-012
n = 13 3.73543414E-007 3.73544102E-007 -6.88273507E-013
n = 14 3.7230068E-007 3.7230081E-007 -1.29984076E-013
n = 15 3.7130358E-007 3.71303801E-007 -2.21193264E-013
n = 16 3.70491398E-007 3.70491787E-007 -3.89377299E-013
n = 17 3.6982016E-007 3.69821496E-007 -1.33573287E-012
n = 18 3.69260128E-007 3.6926165E-007 -1.52216506E-012
n = 19 3.68787997E-007 3.68789176E-007 -1.17903184E-012
n = 20 3.68386948E-007 3.6838674E-007 +2.07588685E-013

The prediction errors on the right now show both positive and negative signs, which is usually the statistical signature that a model is performing as well as possible. If the errors are indeed uncorrelated (these are not), this usually means only "white-noise machine error" is left in the

errors. The sinusoidal pattern in the above errors may be indicative of some type of oscillatory spectroscopic machine error in the data. Whatever the reason for the pattern, the EM Kerr theory here appears to be performing nearly as best it can.

Inclusion of the non-Euclidean relativistic effects clearly shows a better agreement of predictions compared to the observed Balmer data. These results would seem to support the physical presence of stronger relativistic effects in hydrogen, stronger than just Euclidean special. The first jump to spherical Schwarzschild theory shows the general magnitude of these non-Euclidean effects. The electronic Schwarzschild time dilation is (eq. (1) in this paper)

$$\frac{dt}{d\tau} = \left(1 - \left(\frac{v_n}{c}\right)^2 - \frac{r_S}{r_n}\right)^{-1/2}$$

For hydrogen, $r_S \approx 10^{-15}$ m. The ground state radius of hydrogen equals about 10^{-11} m. The ratio $r_S/r_n \approx 10^{-4}$, essentially as significant as introducing Euclidean special relativity itself , since $(v_n/c)^2 \approx 10^{-4}$.

In these non-Euclidean equations, exclusive use of their simplifications to "circular, equatorial" forms has been utilized. This is allowed, even for "P" or "D" of "F", etc., "aspherical" subshells. The reason why, is how "asphericity" is represented in Kerr theory. The Kerr field itself "goes aspherical" when the frame dragging distance a is not zero. Completely conserved circular and equatorial relativistic Kerr geodesics are possible, even in the aspherical field. In classic Sommerfeld theory, with its assumed spherical symmetry of the potential (as in Dirac theory), ashepericity demanded elliptical orbits. But in this generalized orbit theory, it's the field that "goes aspherical" while the orbit itself remains perfectly circular, with conservation of the (shifted) relativistic Kerr total circular orbital energy and angular momentum. Such is the amazing plasticity of a Kerr geometry, and the generalization of the field geometry used for atomic modeling to Kerr seems to produce predictions closer to nature.

Hydrogen's hyperfine splitting can be inserted into the Kerr theory in an easy manner. The (electron spin)-(proton spin) hyperfine orientation delta-energy contributing to the electron's total orbital energy is

$$\Delta E_{hf} = \pi\hbar \left\langle \frac{\hat{A}\hbar}{2\pi} \right\rangle [F(F+1) - I(I+1) - j(j+1)] \qquad (14)$$

where the quantum numbers *F* and *I* are the (Fermi) proton spin related quantum numbers. The value of *I* is always ½, and for *n* = 1,2, *F* can take on the two values of 0 and 1. For hydrogen's ground state (*n* = 1), the value of $< \hat{A}\hbar / 2\pi >$ can be obtained very accurately from experimentation, and equals $< \hat{A}\hbar / 2\pi > \; = 1420.405751768\,\text{Mhz}$

The spin-spin hyperfine delta energy can be easily incorporated in the Kerr theory by simply summing it with the spin-orbit and QED delta energies. The (negative of the) total (sum of the) spin-orbit, QED and spin-spin (hyperfine) delta energy is set equal to the constant β_0 term in solving for the electromagnetic Kerr frame dragging parameter *a*:

$$\beta_0 = -(\Delta E_{so} + \Delta E_{QED} + \Delta E_{hf})$$

Upon root solving, this produces an electromagnetic Kerr frame dragging parameter $a_{n,j,l,s,F,I}$ which is now a function of all of hydrogen's quantum numbers, Dirac and Fermi, and with Euclidean QED effects incorporated.. The hyperfine splittings were programmed into the electromagnetic Kerr transition wavelength predictions, and due to their low value of energy shifts, they did not produce significantly different results as compared to those last reported.

Conclusions and Further Theory

The following conclusion seems inescapable: There exists stronger non-Euclidean relativistic effects in hydrogen as compared to what is predicted by modern Euclidean Dirac theory, even including the effects of the most accurate theory of the atom to date, QED. The better agreement of the predictions by the non-Euclidean Kerr field theory with the observed data suggest the electron is experiencing a greater amount of time dilation as previously thought, and modeled by QED. The increase in time dilation, above and beyond QED Euclidean theory, is due to not only the electron's velocity (*v*), but also its position (*r*) in a generalized atomic-sized electromagnetic Kerr field.

Perhaps the entirety of Euclidean-based QM should be generalized to a more non-Euclidean field theory. The route starts with a generalization of Schroedinger's equation:

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi(r,\theta,\phi) + V(r,v)\psi(r,\theta,\phi) = E(r,v)\psi(r,\theta,\phi) \qquad (15)$$

Dirac inserted the restricted special relativistic form of *E* (whose relativistic kinetic energy is only a function of *v*), and a non-relativistic Coulombic potential *V* (which is only a function of *r*). In (15), the inserted energy functionality (total and potential) on both *r* and *v* written as such, sets up the jump to a more "generalized" geometry, which requires (nonlinear) functionality on

both *r* and *v*. To immediately generalize this differential equation for a bound two-body system such as hydrogen, the total and potential energy equations are identified with the electromagnetic Kerr circular equatorial equations:

$$E(r,v)=\frac{\mu c^2}{2}\left[\left(\left(1-\frac{r_S}{r}+r_S a\frac{v}{r^2}\right)\frac{dt}{d\tau}\right)^2-1\right] \tag{16}$$

and

$$V(r,v)=-\frac{r_S}{2r}+\frac{1}{2}v^2+\frac{1}{2}\left(1-E^2(r,v)\right)\left(1+\frac{a^2}{r^2}\right)-\frac{r_S}{2r^3}\left(vr-aE(r,v)\right)^2 \tag{17}$$

These nonlinear energy equations are obviously more complicated than Euclidean theory (comes with being more "general"), but the tremendous simplicity of their circular equatorial cases should reduce solution complexity. The results would naturally include magnetism as part of the metric structure, per the inclusion of *a* in the equations. Recall in non-Euclidean field theory, magnetism is model by a field warp, not an orbit shape shift, and circular equatorial orbit theory suffices, even when EM Kerr magnetic fields exist. Kerr-Schroedinger circular wave equation solutions could be found, and then Born probability introduced. The generalized expectations should result in equations similar in form and value to the equations already presented here, (Dirac) for spin-orbit magnetic and QED effects. Also, as proven here, this generalization of the "atomic sized" forces in hydrogen produces predictions in better agreement with hydrogen's observed Balmer spectral data.

Appendix

The extended GEM Equivalence Principle (EP) allows setting the electronic force on the electron while in a subshell, equal to any other equal force, inertial or field induced. Bohr used this "extension" of Einstein's EP by equating the electronic force on the electron, to the electron's "inertial" force:

$$\frac{1}{4\pi\varepsilon_0}\frac{|e_e e_p|}{r^2} = m_e a \tag{A.1}$$

The right of (A.1) comes from simply $F = ma$, but this force on the right is actually a field induced force. For extension of the simple Euclidean theory into the general non-Euclidean domain, the GEM EP allows deriving a type of "equivalent gravitational force model" for insertion on the right. The obvious candidate is

$$\frac{1}{4\pi\varepsilon_0}\frac{|e_e e_p|}{r^2} = G\frac{\chi e_p m_e}{r^2} \tag{A.2}$$

The right side of (A.2) is an "effective gravitational force." This effective force exactly equals the electronic force on the left. The "unification curvature parameter" χ is a mass-to-charge ratio which converts the proton's charge e_p into an amount of "effective central mass" which produces exactly the same magnitude of the electronic binding forces on the left of (A.2)

Solving for χ produces

$$\chi = \frac{1}{G4\pi\varepsilon_0}\frac{|e_e|}{m_e} \tag{A.3}$$

In the effective gravitational model, the effective central mass χe_p binds the electron's mass m_e into orbits of the exact same energy magnitude produced by the actual electronic binding forces.

The extension into EM non-Euclidean geometries is made by utilizing the definition of the "gravitational radius," which is the Schwarzschild radius in gravitational Schwarzschild theory. In gravitational theory this radius is $r_S = 2GM / c^2$ where M is the rest mass of the central body.

Inserting the effective central mass $M = \chi e_p$ into this equation produces the electronic Schwarzschild radius, devoid of *G*:

$$r_S = \frac{1}{2\pi\varepsilon_0}\frac{\left|e_e e_p\right|}{m_e c^2} \tag{A.4}$$

This "characteristic/field-defining length" enters the elements of a Schwarzschild metric tensor, producing a tightly-curved, atomic-sized electronic Schwarzschild field.